Communicated by Christoph von der Malsburg

Computing with Arrays of Coupled Oscillators: An Application to Preattentive Texture Discrimination

Pierre Baldi
Jet Propulsion Laboratory and Division of Biology, California Institute of Technology, Pasadena, CA 91125 USA

Ronny Meir
Division of Chemistry, California Institute of Technology, Pasadena, CA 91125 USA

Recent experimental findings (Gray et al. 1989; Eckhorn et al. 1988) seem to indicate that rapid oscillations and phase-lockings of different populations of cortical neurons play an important role in neural computations. In particular, global stimulus properties could be reflected in the correlated firing of spatially distant cells. Here we describe how simple coupled oscillator networks can be used to model the data and to investigate whether useful tasks can be performed by oscillator architectures. A specific demonstration is given for the problem of preattentive texture discrimination. Texture images are convolved with different sets of Gabor filters feeding into several corresponding arrays of coupled oscillators. After a brief transient, the dynamic evolution in the arrays leads to a separation of the textures by a phase labeling mechanism. The importance of noise and of long range connections is briefly discussed.

1 Introduction

Most of the current wave of interest in oscillations and their possible role in neural computations stems from the recent series of experiments by Gray et al. (1989) and Eckhorn et al. (1988). In these experiments, performed on anesthetized and alert (Gray et al. 1989b) cats, moving light bars are presented as visual stimuli and neuronal responses are extracellularly recorded from several electrodes implanted in the first visual cortical areas (mostly area 17). The first observation is that groups of neurons, within a cortical column, tend to engage in stimulus specific oscillatory responses in the 40–60 Hz range. The second and most striking finding is the existence of transient intercolumnar zero phase-locking occurring over distances of several millimeters (at least up to 7 mm) and reflecting global stimulus properties. For instance, elongated or collinear moving light bars of specific orientation elicit zero phase-locked periodic

responses in separated columns with nonoverlapping receptive fields. In contrast, uncorrelated oscillations are observed in the case of similar but noncollinear stimuli.

One possible interpretation of these results has been advanced in the form of the so-called labeling hypothesis. In the labeling hypothesis, temporal characteristics such as the phases (and/or frequencies) of pools of oscillating neurons are used to encode information, in particular to label various features of an object by synchronous activity of the corresponding feature extracting neurons (von der Malsburg 1981). Phase-lockings then serve to link associated features in different parts of the visual field (see also von der Malsburg and Schneider 1986) and, in particular, to represent the coherency of an object. These results and hypothesis suggest that large arrays of coupled oscillating elements, where computations are carried by the transient spatial organization of phase and frequency relationships, may be an important component of neural architectures. If so, one natural area where oscillator networks could be of use is in early vision processing tasks.

Clearly, the two central issues are the origin of the distant synchronizations observed by Gray et al. and whether a useful role for oscillations exists (for instance in the form suggested by the labeling hypothesis), in natural as well as artificial neural systems. In what follows, we first describe how simple coupled oscillator models can be used to account for the experimental data and investigate the more general computational issues. We then apply these concepts to a specific problem in early vision: preattentive texture discrimination. We demonstrate how in a simple oscillator architecture, textures in an image can be separated by the synchronization of populations of oscillators corresponding to each texture.

2 Coupled Oscillator Models

A classical way of modeling coupled oscillators is based on the observation that, once relaxed to its limit cycle, one oscillator can be described by a single parameter: its phase θ_i along the cycle. The behavior of a population of n interacting oscillators can then be approximated by the system

$$\frac{d\theta_i}{dt} = \omega_i(t) + f_i(\theta_1, \ldots, \theta_n) \tag{2.1}$$

where the variables $\omega_i(t)$ represent the internal frequencies and/or the external driving inputs, when their action is independent of the current phases. The functions f_i take into account the coupling among the oscillators assuming that such effects depend only on the phases. The oscillators are located at the vertices of a graph of interactions and, typically, the coupling functions f_i are symmetric of the form $\sum_{j \in V_i} f(\theta_j - \theta_i)$, where

V_i is the set of vertices j adjacent to i and f is an odd periodic function such as $f(\theta) = \sin(\theta)$. Thus, assuming a constant coupling strength, (2.1) typically becomes

$$\frac{d\theta_i}{dt} = \omega_i(t) + K \sum_{j \in V_i} \sin(\theta_j - \theta_i) \tag{2.2}$$

For instance, Cohen et al. (1982) used a one-dimensional version of (2.2) with nearest neighbor coupling to analyze the lamprey locomotion. More general one-dimensional models have been investigated in great detail by Kopell and Ermentrout (1986, 1988), by studying under which conditions their solutions can be approximated by the solutions of the partial differential equation obtained in the continuum limit. When $\omega_i = 0$ (or a constant) and the graph of interactions is the two-dimensional square lattice, (2.2) yields the well known XY model of statistical mechanics (see, for instance, Kosterlitz and Thouless 1973). Fully interconnected versions of (2.2) have also been studied (Kuramoto and Nishikawa 1987). It should be kept in mind that, in general, the coupled limit cycle approach yields satisfactory results provided two conditions are satisfied: (1) the population of nonlinear oscillators should be fairly homogeneous; and (2) the oscillators should not operate in a regime that significantly perturbs the wave form and amplitude of their stable limit cycles, which implies that the coupling strengths (and the external inputs or noise, if any) should not be too strong.

Simulations in two dimensions (and analytical results in one dimension) show that synchronization of oscillators, over distances consistent with the data, seldomly occur if only local nearest neighbor connections are used. To account for the distant phase-lockings, Kammen et al. (1989) assume the existence of a common feedback and consider a comparator model of the form

$$\frac{d\theta_i}{dt} = \omega_i + K \sin(\theta - \theta_i) \tag{2.3}$$

where θ is the average phase $(\sum_i \theta_i / n)$. A different possibility we suggest is to assume that the synchronization of groups of neurons in remote columns rests on the coupling induced by the long horizontal connections, extending over several millimeters, between columns of similar orientation preference (see, for instance, Ts'o et al. 1986). To model the data of Singer and Gray along these lines, consider a caricature of the cortical surface as in Figure 1, where ocular dominance stripes and orientation columns intersect orthogonally. The phase of a neuron or of a group of coherently oscillating neurons within a cortical column is represented by a variable θ_i and satisfies

$$\frac{d\theta_i}{dt} = \omega_i(t) + \eta_i(t) + \sum_{j \in V_i} K_{ji} \sin(\theta_j - \theta_i) \tag{2.4}$$

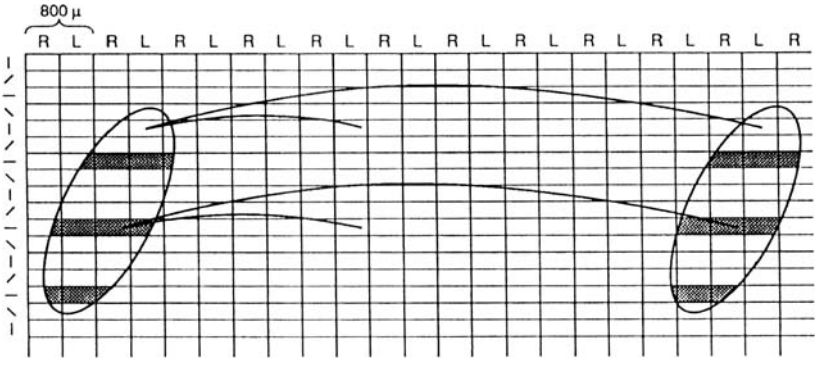


Figure 1: A caricature of the cortical surface (area 17), with two excited regions, one for each stimulating horizontal bar, and showing the ocular dominance columns intersecting the orientation stripes orthogonally. Long-range connections, over several millimeters, between columns with similar orientation are indicated. The combined receptive fields of the cells within one of the excited regions cover exactly the region of space corresponding to one of the moving bars. Within an excited region, activity is maximal for the cells with optimal orientation (dark stripes).

where $\eta_i(t)$ is gaussian noise with a fixed variance. The frequency ω is zero except in the regions that have been activated by the stimulus. For an excited region, corresponding to several columns with combined receptive fields covering the stimulus, the frequency ω is set to some constant value. Now, within such an oscillating region, neural activity is variable, maximal for neurons corresponding to the optimal orientation and almost nonexistent for oscillators associated with the orthogonal direction (i.e., for columns where, most likely, the oscillation is mainly subthreshold). This aspect can be modeled by introducing an amplitude $A_i(t)$, which is zero everywhere except in an excited region where, to a first approximation,

$$A_i(t) = \begin{cases} a & \text{if } i \text{ has optimal or near optimal orientation} \\ b & \text{if the oscillations at } i \text{ are mostly subthreshold} \end{cases} \tag{2.5}$$

with $a > b > 0$ (of couse, one can consider a continuum of amplitudes). Finally, for the coupling strengths K_{ij}, different cases would need to be considered depending on the distance between i and j and also the amplitude of the activity at these sites. From a formal standpoint, such a description is only a discretized version of a model shown to fit the data by Sompolinsky et al. (1990) (although their interpretation of some

of the variables seems to differ), which can be summarized by (2.4) and two additional equations.

$$A_i(t) = \begin{cases} A(\Delta\theta) & \text{in an excited region} \\ 0 & \text{otherwise} \end{cases} \tag{2.6}$$

where $\Delta\theta$ represents the difference between the preferred orientation at i and the orientation of the local stimulus and A is, for instance, a gaussian function. In addition,

$$K_{ij}(t) = A_i(t)A_j(t)K[d(i,j)] \tag{2.7}$$

where $K[d(i,j)]$ takes into account all the connections: from the very short, presumably roughly isotropic, to the long ones, extending over several millimeters and dependent on similarity of orientation. Notice that (2.7) does not necessarily require to postulate the existence of fast hebbian synapses. It only models the coupling strength between two oscillators as being dependent on the level of activity present in the two neural regions they represent. Finally, it appears from anatomical and physiological studies that the excitation due to intracortical connections by far exceeds the excitation mediated by thalamic afferents corresponding to the inputs. Thus, if (2.4) is to be used as a cortical model, the parameters should be chosen so that the dominant contribution arises from the coupling terms.

Synchronization issues in neural networks have also been addressed in Atiya and Baldi (1990) using continuous analog and integrate and fire model neurons and in Wilson and Bower (1990) using more detailed compartment models. In Lytton and Sejnowski (1990), compartment models are also used to demonstrate the synchronizing action that inhibitory interneurons such as basket or chandelier cells may have on cortical pyramidal cells. Although more experimental data are critically needed, a consensus seems to emerge that phase-locking of remote groups of neurons results either from common feedback projections, either from the action of long-range horizontal connections, or, of course, from various possible combinations of both mechanisms. In any event, the experimental data suggest that it may be worth investigating the properties of a new type of "neural" network, consisting of large arrays of coupled oscillators where computations are carried through rapid spatiotemporal self-organization of coherent regions of activity. The coupled oscillator models described here are extremely simplistic and are not meant to closely fit what is presently known of cortical neuroanatomy and neurophysiology. Rather, a minimal set of assumptions is introduced and plenty of room exists for successively incorporating more realistic details and manipulating more complex dynamics. One advantage of the relative simplicity of this approach is that fairly large systems can be simulated on a digital computer. Since several models have already been proposed, it becomes even more important to try to assess whether there are any intrinsic computational advantages to oscillator networks and

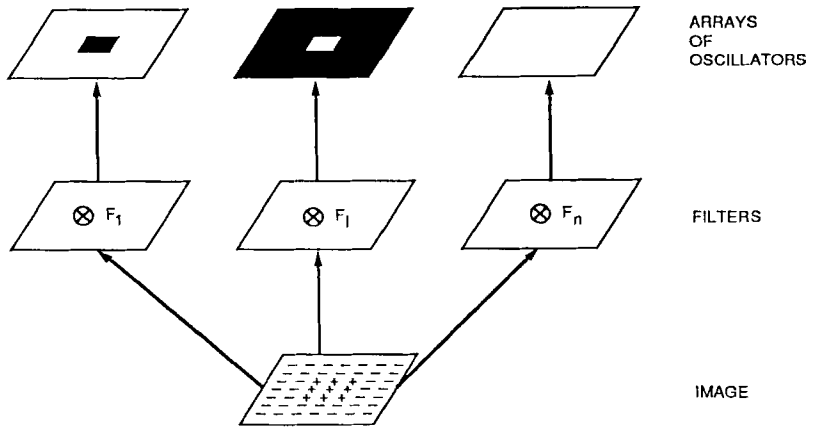


Figure 2: The basic architecture: the image is first convolved with several banks of Gabor filters, which, in turn, excite several arrays of coupled oscillators.

whether they could be used for certain specific tasks. A possible natural domain of application for networks of oscillators seems to be early vision. As a specific example, we have attempted to apply the previous concepts to the problem of preattentive texture discrimination.

3 Texture Discrimination

The real world is seldom constructed of homogeneous objects whose boundaries are given by luminosity gradients. Many natural images contain regions composed of different microfeatures (texture elements) that repeat in some quasiperiodic manner to cover the surface. Examples of textured surfaces could be fabric, lawn, water bubbles, etc. In this context, it would be of interest to understand how the visual system manages to find boundaries between objects, where no luminosity gradient exists. Over the past 15 years or so, much work in early vision has focused on how regions composed of different texture elements are segmented. In particular, Julesz (see, for example, Julesz 1984) has investigated several classes of artificial textures and found that certain pairs of textures can be preattentively discriminated, while others require serial search. Although the sharp division between preattentive and attentive texture segmentation has been questioned in recent years (Gurnsey and Browse 1987), it seems that many texture discrimination problems are indeed low level (i.e., parallel and bottom up).

A few algorithms have been described in the literature that seem to achieve texture discrimination abilities similar to those of humans. Two recent contributions are by Fogel and Sagi (1989) and Malik and Perona (1990). These algorithms are often motivated by analogies with the known neurophysiology (see Van Essen et al. 1989 and references therein). In particular, the first operation carried out in layer IV of primary visual cortex is believed to consist in part of a filtering of the image through a set of feature detectors of varying orientations and spatial frequencies. It would thus be reasonable to surmise that this initial filtering process is a necessary requirement for any model attempting biological plausibility. Since there is little experimental data to guide us in understanding what the visual system does with this raw filtered image, most current

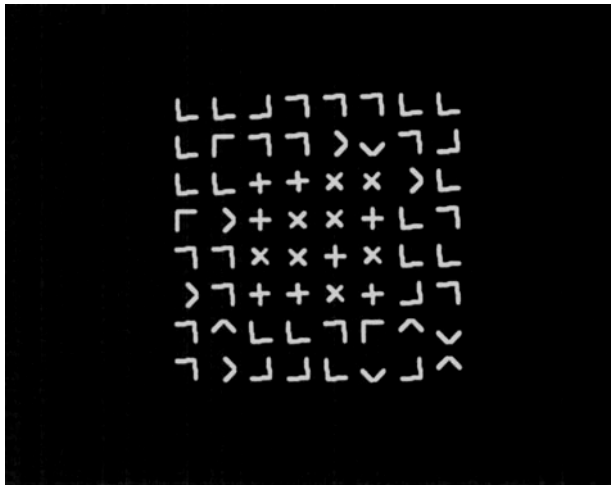
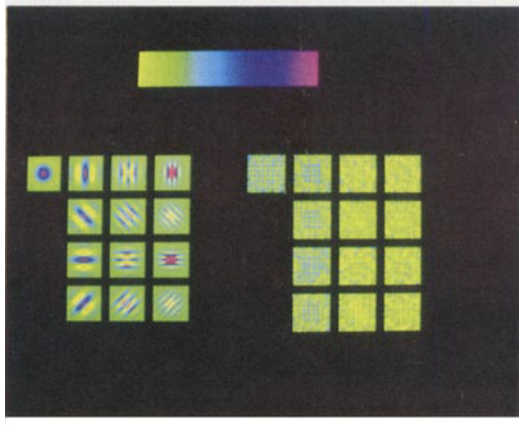
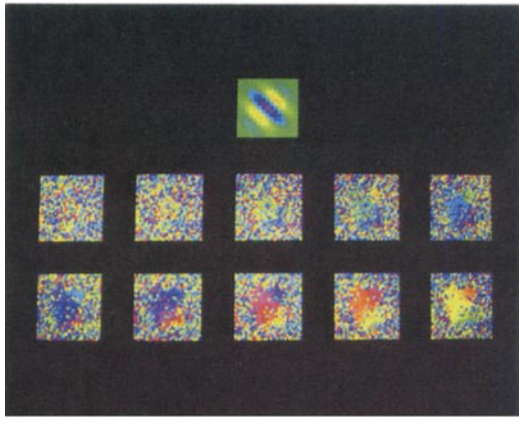


Figure 3: (a) An example of a 64×64 texture used in our simulations. The size of the microfeatures is 4×4 pixels. (b) [*facing page*] The receptive fields of 13 different even filters are represented on the left. On the right, the energies associated with the corresponding convolutions with the texture of a. Intensity levels are coded by colors. The first filter (top left) is just a laplacian operator. The coefficients in equation 3.1 used to generate the remaining 12 Gabor filters are $\sigma = 3$, $\nu = 2\pi n/4$ with $n = 0.6$, 1.2, and 1.8, and $\alpha = 0$, $\pi/4$, $\pi/2$, $3\pi/4$, π.

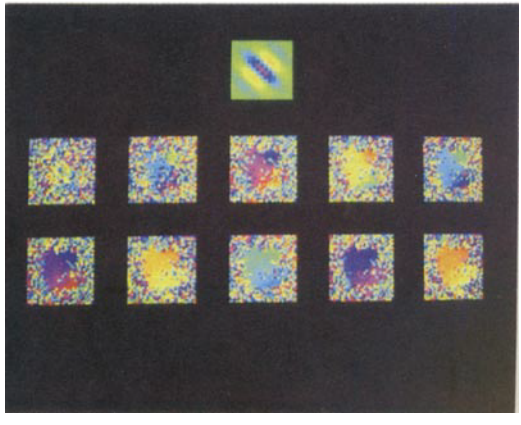
Figure 4: *Facing page.* (a) The temporal evolution of an oscillator array corresponding to one type of filter (shown at the top). Phases are coded by colors and the entire sequence corresponds to 2 oscillator cycles. Parameters, for all the simulations, are $\omega = 2$, $[E(\eta_i^2)]^{1/2} = 0.4$, $K = 60$, and $T = 0.02$, and boundary conditions are free. (b) Same as a, but on a slower time scale. The entire sequence corresponds to 5 oscillator cycles.



3(b)



4(a)



4(b)

algorithms diverge at this point. Assuming that the textured image consists of a pair of preattentively discriminable textures (such as in Fig. 3a), an early visual channel must exist that can discriminate between the two. The problem faced by any system at this stage then, is to use the biased response of at least one of the filters to segment the image quickly and robustly. Fogel and Sagi (1989), for instance, proposed first smoothing the filtered image with a gaussian filter, applying some noise reduction techniques, and then using a laplacian operator to detect the boundaries. Malik and Perona (1990) used a half-wave rectification stage combined with lateral inhibition between different filters, and demonstrated its effectiveness in reproducing most of the known psychophysical data. In what follows, we address the problems of smoothing noisy response profiles and enhancing boundary formations between regions corresponding to different textures using oscillator architectures. Our main goal is to exploit the *dynamics* of the system to perform the computational task, without recourse to further filtering and smoothing operations which, at this stage, have not been found. Thus, although we apply our ideas to texture segmentation, we are mainly concerned with the dynamic aspects of such processing.

We have considered simple but nontrivial texture images, of size 64×64 (see Figs. 3 and 5), constructed so as to avoid differences in luminosity among texture patterns. As already mentioned, oscillator arrays cannot be used alone but only as a module of a more complex system, a few processing stages away from the sensory interface. Thus, the images are first convolved with several 32×32 banks of Gabor filters (see, for instance, Daugman 1985) of even and odd symmetry with different orientations and spatial frequencies. A filter with coordinate center (a, b), spatial frequency ν, width σ, and orientation α can be described by the convolution kernel

$$e^{-[(x-a)^2+(y-b)^2]/2\sigma^2} \sin[\nu(x-a)\cos\alpha - \nu(y-b)\sin\alpha + \phi] \tag{3.1}$$

The responses of the odd ($\phi = 0$) and even ($\phi = \pi/2$) filters are squared and summed to give the energy at each orientation and spatial frequency (see Fogel and Sagi 1989). We emphasize that our aim is not to find the best filters nor the most biologically plausible representation for them, but in the dynamic segmentation of the image. Our algorithm differs from currently existing ones by feeding the response of each bank of filters into a corresponding 32×32 array of oscillators satisfying equations (2.4) and (2.7) (see Figs. 2, 3b and 5b). The basic idea is to use the phase-locking properties of long range coupled oscillators (Kuramoto and Nishikawa 1987) to achieve separation in *time* between the figure (one type of texture) and the ground (the other texture). Moreover, as is demonstrated in Figures 5 and 6, the same approach can be used to detect signals in noisy environments in a very fast and parallel fashion. Here, the amplitude $A_i(t)$, in a given array, is equal to the energy of the corresponding pair of Gabor filters and does not vary with time. The frequency ω is chosen

to be constant and identical for all the oscillators, and η_i are independent identical gaussian random variables. The connection strength pattern $K[d(i,j)]$ is zero everywhere except for two oscillators i and j belonging to the same array and contained in a square with side of length 8, in which case the coupling assumes a constant value K. In particular, at this stage, there are no connections between oscillators pertaining to different arrays associated with different banks of Gabor filters. In simulations, we have found that the prescription for the couplings K_{ij} given by (2.7) is often insufficient to generate reasonable phase boundaries and therefore we have replaced it by the more general form

$$K_{ij} = A_i A_j K[d(i,j)] F(\bar{A}_i - T) F(\bar{A}_j - T) \tag{3.2}$$

where T is some threshold value, $\bar{A}_i$ is the average of the activity in a neighborhood of i (taken here to be a square with side of length 5), and $F(x) = 1$ if $x > 0$ and 0 otherwise.

Examples of the evolution, from a random initial state, of the phases of the oscillators in some of the arrays are shown in Figures 4 and 6. It can be seen that very rapidly, within a few oscillator cycles (typically 2 to 4), the figure texture oscillates coherently (against a random background) in one of the arrays. The separation between the regions is certainly not sharp. However, precise boundaries should not be expected in preattentive vision. Furthermore, sharper contours could easily be achieved by introducing additional mechanisms such as fast hebbian synapses coupled with the oscillator dynamics or finer threshold adjustments. No attempt has been made here to optimize any of the parameters or the connectivity. This work is intended only as a demonstration and many refinements seem possible. In particular, interactions between the different filters and/or oscillators should be investigated. Similarly, only a restricted set of textures has been generated for this study.

In conclusion, a demonstration has been given that, at least in principle, preattentive texture segmentation can be solved by a temporal phase coherence mechanism and implemented with simple arrays of coupled oscillators. The basic components of our algorithm are the phase-locking of strongly excited, distantly coupled oscillators corresponding to the figure and the decoupling of weakly excited, weakly coupled oscillators corresponding to the ground. Two critical parameters should be emphasized: the extent of the lateral connections and the active role played by the noise. In simulations of the XY model with nearest neighbor couplings, we have observed that, starting from a random initial configuration, these systems tend to rapidly organize themselves into a patchy structure of phase-locked regions, a few oscillators in diameter or so. This in turn suggests that, if synchronizations are to be achieved over significantly greater distances, longer connections become a necessity. Analytically, it can be shown that fully interconnected systems tend to synchronize below a certain critical temperature (see Kuramoto and Nishikawa 1987). In the case of visual images, texture regions naturally

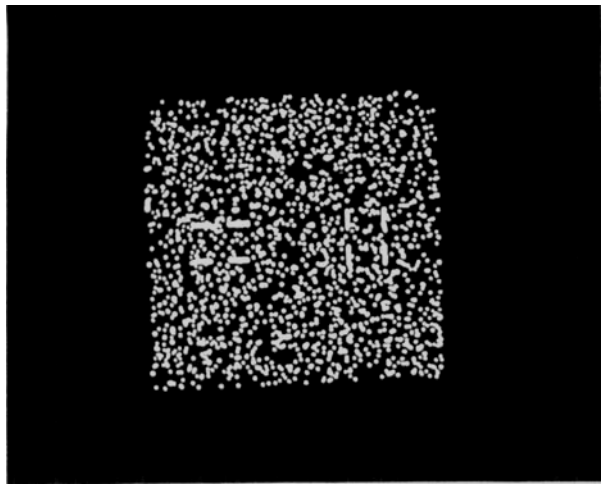


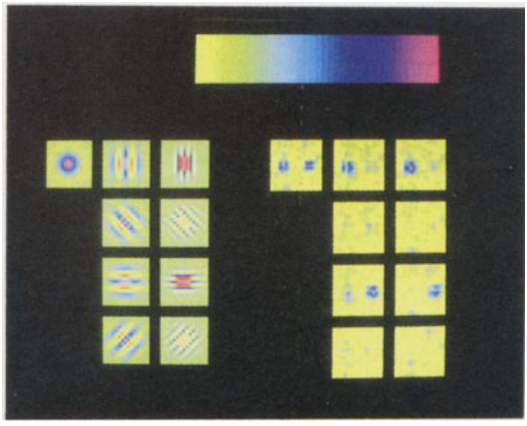
Figure 5: (a) Similar to Figure 3a, but with a different image. Only 9 filters are shown.

cover several degrees of visual field corresponding to several millimeters of primary visual cortex [the magnification factor, at excentricity E, is roughly given by $M = (0.8 + E)^{-1.1}$ mm/deg (Van Essen et al. 1984)]. This is not inconsistent with the anatomical evidence for the existence of long-range connections within cortical areas and the fact that, in simulations, couplings over ranges roughly comparable to the size of a typical figure are needed for phase-locking of the oscillators within the figure region. The noise also plays an important role: it has a small effect on the figure where the effective couplings are large but is essential in decoupling the background by randomizing the phases. The same principles used here could perhaps be applied to other problems in vision such as, for instance, illusory conjunctions and contour filling type of experiments.

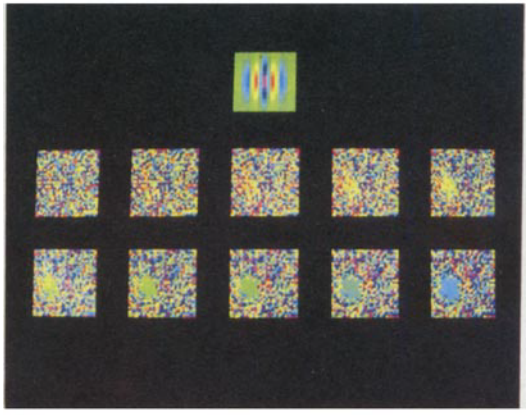
It is too early and beyond the scope of this article to compare the performance of our algorithm to others or to try to match it with the available psychophysical data. The mechanisms described should be refined and

Figure 5: *Facing page.* (b) Similar to Figure 3b, but with a different image. Only 9 filters are shown.

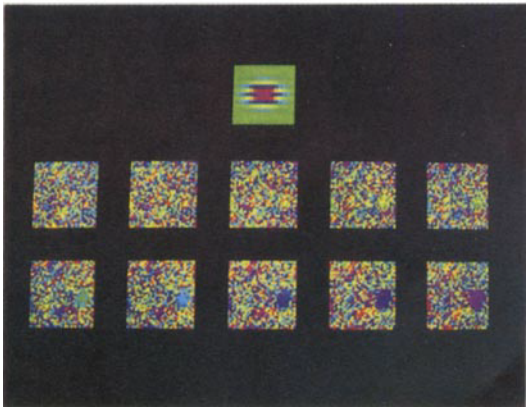
Figure 6: *Facing page.* (a,b) Evolution of two arrays of oscillators corresponding to two different filters shown at the top of each picture. Each sequence corresponds to 2 oscillator cycles.



5(b)



6(a)



6(b)

tested on a variety of different situations. Similarly, it is premature to try to assess whether oscillator architectures present any intrinsic computational advantages. Certainly, oscillator networks have remarkable properties of robust and very rapid self-organization and naturally utilize the temporal dimension. Hardwired implementations of coupled oscillators do not seem to pose, at least in principle, any conceptual obstacles. On the other hand, the same tasks described here could be achieved with stationary signals and it should be kept in mind that little information per se is contained in a group of coherently firing neurons. If coupled oscillators are to be used in a computation, significant additional machinery is required to process and route the information.

Acknowledgments

This work is supported by a McDonnell-Pew Grant and ONR Contract NAS7-100/918 to P. B., ONR Grant N00014-87-K-0377 (J. J. Hopfield, P. I.), and a Weizmann Fellowship to R. M.

References

Atiya, A., and Baldi, P. 1989. Oscillations and synchronizations in neural networks. An exploration of the labeling hypothesis. *Int. J. Neural Syst.* **1**(2), 103–124.

Cohen, A. H., Holmes, P. J., and Rand, R. H. 1982. The nature of the coupling between segmental oscillators of the lamprey spinal generator for locomotion: A mathematical model. *J. Math. Biol.* **13**, 345–369.

Daugman, J. G. 1985. Uncertainty relations for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters. *J. Opt. Soc. Am.* **2**(7), 1160–1169.

Eckhorn, R., Bauer, R., Jordan, W., Brosch, M., Kruse, W., Munk, M., and Reitboek, H. J. 1988. Coherent oscillations: A mechanism of feature linking in the visual cortex? *Biol. Cybernet.* **60**, 121–130.

Fogel, I., and Sagi, D. 1989. Gabor filters as texture discriminators. *Biol. Cybernet.* **61**, 103–113.

Gray, C. M., König, P., Engel, A. K., and Singer, W. 1989a. Oscillatory responses in cat visual cortex exhibit inter-columnar synchronization which reflects global stimulus properties. *Nature (London)* **338**, 334–337.

Gray, C. M., Raether, A., and Singer, W. 1989b. Stimulus-specific intercolumnar interactions of oscillatory neuronal responses in the visual cortex of alert cats. 19th Neuroscience Meeting, Phoenix, Arizona (Abstract).

Gurnsey, R., and Browse, R. A. 1987. Aspects of texture discrimination. In *Computational Processes in Human Vision*, Z. Pylyshyn, ed., Ablex, New York.

Julesz, B. 1984. Towards an axiomatic theory of preattentive vision. In *Dynamic Aspects of Neocortical Function*, G. Edelman, W. Einer, and W. Cowan, eds. John Wiley, New York.

Kammen, D. M., Holmes, P. J., and Koch C. 1989. Cortical architecture and oscillations in neuronal networks: Feed-back versus local coupling. In *Models of Brain Function*, R. M. J. Cotterill, ed. Cambridge University Press, Cambridge, pp. 273–284.

Kopell, N., and Ermentrout, G. B. 1986. Symmetry and phaselocking in chains of weakly coupled oscillators. *Commun. Pure Appl. Math.* **XXXIX**, 623–660.

Kopell, N., and Ermentrout, G. B. 1988. Coupled oscillators and the design of central pattern generators. *Math. Biosci.* **90**, 87–109.

Kosterlitz, J. M., and Thouless, D. J. 1973. Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys. C* **6**, 1181–1203.

Kuramoto, Y., and Nishikawa, I. 1987. Statistical macrodynamics of large dynamical systems. Case of a phase transition in oscillator communities. *J. Stat. Phys.* **49**(3/4), 569–605.

Lytton, W. W., and Sejnowski, T. J. 1990. Inhibitory interneurons may help synchronize oscillations in cortical pyramidal neurons. Presented at the 1990 Conference on Neural Networks for Computing, Snowbird, Utah.

Malik, J., and Perona, P. 1990. Preattentive texture discrimination with early vision mechanisms. *J. Opt. Soc. Am. A*, **7**(5), 923–932.

Sompolinsky, H., Golomb, D., and Kleinfeld, D. 1990. Global processing of visual stimuli in a neural network of coupled oscillators. Preprint.

Ts'o, D. Y., Gilbert, C. D., and Wiesel, T. N. 1986. Relationships between horizontal interactions and functional architecture in cat striate cortex as revealed by cross-correlation analysis. *J. Neurosci.* **6**(4) 1160–1170.

Van Essen, D. C., Newsome, W. T., and Maunsell, J. H. R. 1984. The visual field representation in striate cortex of the macaque monkey: Asymmetries, anisotrophies and individual variability. *Vision Res.* **24**, 429–448.

Van Essen, D. C., DeYoe, E. A., Olavarria, J. F., Knierim, J. J., Fox, J. J., Sagi, D., and Julesz, B. 1989. Neural responses to static and moving texture patterns in visual cortex of the macaque monkey. In *Neural mechanisms of Visual Perception. Proceedings of the Retina Research Foundation Symposium*, Vol. 2, D. M. Lam and C. Gilbert, eds. Portfolio Publishing, Woodlands, TX.

von der Malsburg, C. 1981. The correlation theory of brain function. Internal Report 81-2, Department of Neurobiology, Max Planck Institute for Biophysical Chemistry.

von der Malsburg, C., and Schneider, W. 1986. A neural cocktail-party processor. *Biol. Cybernet.* **54**, 29–40.

Wilson, M. A., and Bower, J. M. 1990. Computer simulation of oscillatory behavior in cerebral cortical networks. Proceedings of the 1989 NIPS Conference, Denver, Colorado. In *Advances in Neural Information Processing Systems 2*, edited by David S. Touretzky, Morgan Kaufmann, pp. 84–91.

Received 18 April 90; accepted 3 August 90.